OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2607

Mechanics 1

Friday **14 JANUARY 2005** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8$ m s^{-2} unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

Registered Charity 1066969
[Turn over

1 **(a) In this part-question take g as $10\,\mathrm{m\,s^{-2}}$.**

A small ball is released from rest. It falls for 2 seconds and is then brought to rest over the next 5 seconds. This motion is modelled in the speed-time graph Fig. 1.

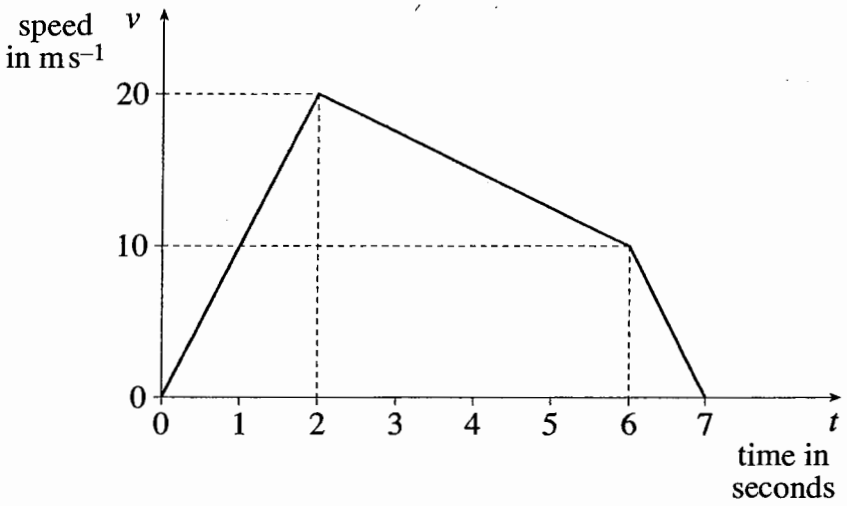


Fig. 1

(i) Calculate the distance fallen from $t = 0$ to $t = 7$. [3]

(ii) Find the acceleration of the ball from $t = 2$ to $t = 6$, specifying the direction. [3]

(iii) Obtain an expression in terms of t for the downward speed of the ball from $t = 2$ to $t = 6$. [3]

(iv) State the assumption that has been made about the resistance to motion from $t = 0$ to $t = 2$. [1]

(b) The position vector, $\mathbf{r}$, of a particle at time t is given by

$$\mathbf{r} = t^2\mathbf{i} + (5t - 2t^2)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors, lengths are in metres and time is in seconds.

(i) Find an expression for the acceleration of the particle. [4]

(ii) Is the particle ever at rest? [2]

[Total 16]

2 A small box B of weight 400 N is held in equilibrium by two light strings AB and BC. The string BC is fixed at C. The end A of string AB is fixed so that AB is at an angle α to the vertical where $\alpha < 60°$. String BC is at 60° to the vertical. This information is shown in Fig. 2.

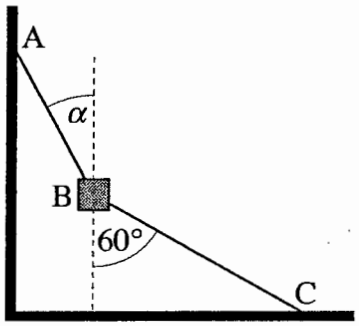


Fig. 2

(i) Draw a labelled diagram showing all the forces acting on the box. [1]

(ii) In one situation, the tension in the string BC is 200 N.

By resolving horizontally and vertically, or otherwise, calculate α and the tension in string AB. [7]

(iii) In a new situation, string AB is fixed so that $\alpha = 30°$ and BC remains at 60° to the vertical.

By drawing a triangle of forces, or otherwise, calculate the tension in the string BC and the tension in the string AB. [4]

(iv) Show carefully, but briefly, that the box cannot be in equilibrium if $\alpha = 60°$ and BC remains at 60° to the vertical. [2]

[Total 14]

[Turn over

3 **(a)** A particle of mass 4 kg is subject to a force **F** and a force $12\mathbf{j}$ N. It has an acceleration of $(2\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-2}}$. Find **F**. [3]

(b) Two blocks are connected by a light inextensible string AC that passes over a small, smooth pulley at B, as shown in Fig. 3. The blocks slide on the horizontal plane and on the inclined plane. The masses of the blocks, the resistances to motion of 4 N and 3 N, and the angle of the inclined plane are also given in the figure. The string sections AB and BC are parallel to the horizontal and inclined planes respectively.

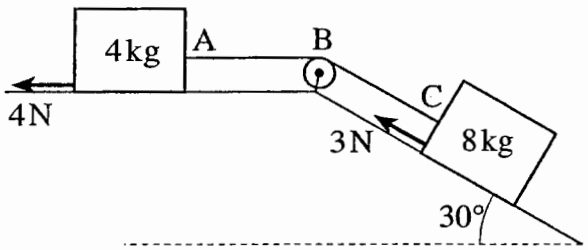


Fig. 3

(i) State what information in the question tells you that

(*A*) the magnitudes of the accelerations of the blocks are the same,

(*B*) the tension in string section AB is the same as that in BC. [2]

The tension in the string is TN and the magnitude of the acceleration of the two blocks is $a\,\mathrm{m\,s^{-2}}$.

(ii) By considering the motion of the 4 kg block, write down an equation connecting T and a. [2]

(iii) Draw a diagram showing the forces acting on the 8 kg block. By considering the motion of this block find another equation connecting T and a. [5]

(iv) Find the value of a. [3]

[Total 15]

4 The trajectory ABCD of a small stone moving with negligible air resistance is shown in Fig. 4. AD is horizontal and BC is parallel to AD.

The stone is projected from A with speed $40\,\mathrm{m\,s^{-1}}$ at 50° to the horizontal.

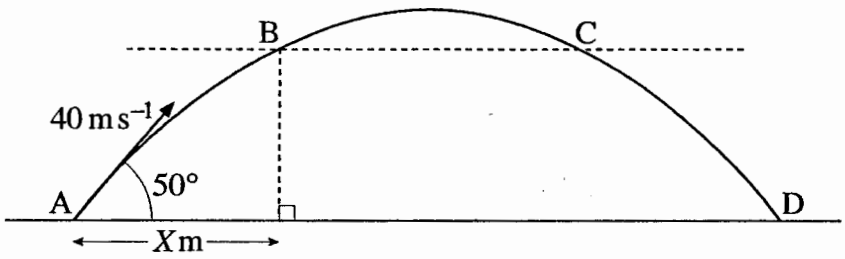


Fig. 4

(i) Write down an expression for the horizontal displacement from A of the stone t seconds after projection. Write down also an expression for the vertical displacement at time t. [3]

(ii) Show that the stone takes 6.253 seconds (to three decimal places) to travel from A to D. [3]

You are given that $X = 30$.

(iii) Calculate the time it takes the stone to reach B. Hence determine the time for it to travel from A to C. [4]

(iv) Calculate the direction of the motion of the stone at C. [5]

[Total 15]

Mark Scheme

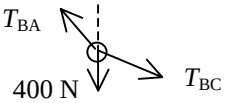
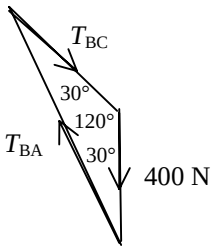
Paper 2607	Name Mechanics 1	Session Jan	Year 2005	Draft final

Solutions and mark scheme

Q 1		mark		
(a) (i)	Area under curve $0.5\times 2\times 20 + 0.5\times(20+10)\times 4 + 0.5\times 10\times 1$ = 85 m	M1 B1 A1	Attempt to find any area under curve or use const accn results Any area correct (Accept 20 or 60 or 5 without explanation) cao	3
(ii)	$\frac{20-10}{4} = 2.5$ upwards	M1 A1 B1	$\Delta v / \Delta t$ accept ± 2.5 Accept – 2.5 downwards (allow direction specified by diagram etc). Accept 'opposite direction to motion'.	3
(iii)	$v = -2.5t + c$ v = 20 when t = 2 $v = -2.5t + 25$	M1 M1 A1	Allow **their** a in the form $v = \pm at + c$ or $v = \pm a(t-2) + c$ cao [Allow $v = 20 - 2.5(t-2)$] [Allow 2/3 for different variable to t used, e.g. x. Allow any variable name for speed]	3
(iv)	Falling with negligible resistance	E1	Accept 'zero resistance', or 'no resistance' seen.	1
(b) (i)	Differentiate **r** **v** = 2t **i** + (5 – 4t) **j** differentiate **v** **a** = 2 **i** – 4 **j**	M1 A1 M1 F1	At least 1 cpt correct. Award for RHS seen Do not award if **i** and **j** lost in **v**. At least 1 cpt correct. FT from their 2 component **v**	4
(ii)	Need **i** and **j** cpts of **v** both zero **i** cpt 0 when t = 0 **j** cpt 0 when t = 1.25 so no	M1 E1	Finding when either cpt of **v** is zero (FT their **v** from (i)) Comparison of times or showing **v** is non-zero for either time.	2
		16	total	

Paper 2607	Name Mechanics 1	Session Jan	Year 2005	Draft final

Solutions and mark scheme

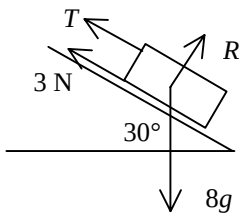
Q 2		mark		sub
(i)		B1	Different labels. All forces present with arrows in correct directions. Condone no angles. No extra forces.	1
(ii)	$T_{BA}\cos\alpha = 400 + 200\cos 60 = 500$	M1	Attempt at resolution vertically. All forces present and at least one resolution attempted. No extra forces. Allow *W* used in place of 400.	
		A1		
	$T_{BA}\sin\alpha = 200\sin 60$	M1	Attempt at resolution horizontally. Both forces present and resolved, one resolution correct. No extra forces.	
		A1		
	Divide $\dfrac{T_{BA}\sin\alpha}{T_{BA}\cos\alpha} = \dfrac{200\sin 60}{500}$	M1	Attempt to eliminate *T* or α	
	so $\tan\alpha = \sqrt{3}/5$ and $\alpha = 19.1066..$			
	so 19.1 (3 s. f.)	A1	Any reasonable accuracy	
	Tension is $100\sqrt{28}$ N (529 N (3 s. f.))	F1	Any reasonable accuracy. FT either order in their equation [If force triangle: M1, A1 2 correct angles (ignore arrows), A1 triangle all correct, M1 A1 for *T*, M1 F1 for angle; if right angled triangle award 0. If Lami: M1 first pair of equations in correct format (condone wrong angles) A1. M1 second pair in correct format (condone wrong angles) A1. M1 Method for solving. A1. F1 FT their first answer if necessary.]	7
(iii)	Using triangle of forces	M1	Attempt at triangle of forces. Ignore angles and arrows. Accept 90, 60, 30 triangle.	
		B1	Triangle, arrows, labels and angles correct	
	Triangle isosceles so tension in BC is 400 N	A1	cao	
	Tension in BA is $2\times 400\times\cos 30 = 400\sqrt{3}$ N (693 N, (3 s. f.))	F1	FT BC only. [If resolution used, M1 for 1 equn; M1 for 2nd equn + attempt to elim; A1; F1. For M marks all forces present but allow $s \leftrightarrow c$ and sign errors. No extra forces. If Lami used: M1 first pair of equations in correct format, condone wrong angles. A1. M1 second pair in correct format, with **correct** angles.F1 FT their first answer if necessary.]	4

Solutions and mark scheme

Q2				
(iv)	Resolve at B perpendicular to the line ABC Weight has unbalanced component in this direction	E1 E1	Attempt to argue unbalanced force Complete, convincing argument. [**or** Resolve horiz and establish tensions equal E1 Resolve vert to show inconsistency. E1]	2
			total	14

Paper 2607	Name Mechanics 1	Session Jan	Year 2005	Draft final

Solutions and mark scheme

Q 3		mark		sub
(a)	$\mathbf{F} + 12\,\mathbf{j} = 4(2\,\mathbf{i} - 4\,\mathbf{j})$	M1	N2L. Allow $\mathbf{F} = m\mathbf{g}$ **a.** No extra forces. Allow 12**j** omitted.	
		A1	Allow wrong signs otherwise correct.	
	$\mathbf{F} = 8\,\mathbf{i} - 28\mathbf{j}$	A1	cao	3
(b) (i)	String is inextensible	E1		
	Pulley is smooth (and string is light)	E1	Accept just 'pulley smooth'.	2
(ii)	N2L $\rightarrow$ $T - 4 = 4a$	M1	N2L. May be $F = \pm mga$. All forces present. No extra forces. Condone sign errors.	
		A1	Any form	2
(iii)		B1	All forces present (ignore R), labelled and with arrows. Accept w, mg etc. Angle not required. No extra forces.	
	N2L down the slope $8g\sin 30 - T - 3 = 8a$	M1	Use of $F = ma$ down the slope with a and T. Accept 3 N omitted. Accept any attempt at weight cpt, but not 8g alone.	
		B1	$8g\sin 30$ correct and seen in an equation.	
		A1	All forces present and correct. Allow sign errors.	
	so $4g - T - 3 = 8a$	A1	Any form but sign of a must be consistent with (ii).	5
(iv)	Eliminate T	M1	Attempt to eliminate leading to a value of a.	
	$4g - 4 - 4a - 3 = 8a$	A1	Elimination correct. FT their 2 equations in 2 unknowns.	
	so $a = 2.68\dot{3}$ so 2.68 (3 s. f.)	A1	cao	3
			total	15

Paper 2607	Name Mechanics 1	Session Jan	Year 2005	Draft final

Solutions and mark scheme

Q 4		mark		sub
(i)	Horiz $(40\cos 50)t$ Vert $(40\sin 50)t - 4.9t^2$	B1 M1 A1	Use of $s = ut + 0.5at^2$ with $a = \pm 9.8$ or ± 10. Allow u = 40. Condone s $\leftrightarrow$ c Any form	3
(ii)	Need $(40\sin 50)t - 4.9t^2 = 0$ so $t = \dfrac{40\sin 50}{4.9}$ = 6.2534… so 6.253 s (3 d. p.)	M1 M1 E1	Equating **their** y to zero. Allow quadratic y only Dep on 1st M1. Attempt to solve. Clearly shown [**or** M1 (allow u = 40 and s $\leftrightarrow$ c) A1 time to greatest height; E1]	3
(iii)	Time AB is given by $(40\cos 50)T = 30$ so T = 1.16679… so 1.17 s **then** **either** By symmetry, time AC is time AD – time AB so time AC is 6.2534… $-\dfrac{30}{40\cos 50}$ = 5.086…. so 5.09 s (3 s. f.) **or** height is $(40\sin 50)T - 4.9T^2$ and we need $(40\sin 50)t - 4.9t^2 = (40\sin 50)T - 4.9T^2$ solved for larger root i.e. solve $4.9t^2 - (40\sin 50)t + 29.08712... = 0$ for larger root giving 5.086…	M1 A1 M1 A1 M1 A1	Equating **their** linear x to 30. Symmetry need not be explicit. Method may be implied. Any valid method using symmetry. cao Complete method to find time to second occasion at that height cao	4
(iv)	$\dot{x} = 40\cos 50$ $\dot{y} = 40\sin 50 - 9.8\times 5.086...$ Need $\arctan\dfrac{\dot{y}}{\dot{x}}$ So –36.761…° so 36.8° below horizontal (3 s.f.)	B1 M1 A1 M1 A1	Must be part of a method using velocities. Use of vert cpt of vel Allow only sign error. FT use of **their** 5.086.. May be implied. Accept $\arctan\dfrac{\dot{x}}{\dot{y}}$ but not use of $\dot{y}_0$. Accept ±36.8 or equivalent. Condone direction not clear.	5
				15

Examiner’s Report